Game Theoretic Analysis of the Multi-Organization Scheduling Problem

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Workshop on Algorithms and Techniques for Scheduling on Clusters and Grids
Outline

1. Motivation
2. The Multi-Organization Scheduling Problem
3. Game-theoretic Model
4. Future Work
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3. Game-theoretic Model
4. Future Work
The importance of cooperation

Current global computing technology (e.g. grid computing systems) makes very clear the importance of creating coalitions of computational resources.
Motivation

Goal: encourage collaboration

If each organization cooperates unconditionally, we can achieve the best utilization possible of the available resources.
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Although (if you look closely) sometimes some concessions must be made:

- $C_{\max}$ that $O^{(1)}$ can achieve by itself: \textbf{1}
- $C_{\max}$ of $O^{(1)}$ in the global optimum configuration: \textbf{2}
Goal: encourage collaboration

What if we have only *selfish* organizations with specific performance goals?

- An organization could just leave the coalition and do all the work by itself instead of helping others (which is even worse for the entire community).

Our goal is to provide a scheduling mechanism that can improve the global performance of the system while assuring that the local performance of each organization will not be penalized for cooperating with others.
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The problem

The multi-organization scheduling problem can be defined as the problem of minimizing the maximum completion time (makespan) of all jobs and, at the same time, minimize locally:

- the makespan of $k$ organizations
  \[ \text{MOSP}(k : C_{\text{max}}) \]

- the average completion time of $k$ organizations
  \[ \text{MOSP}(k : \sum C_i) \]

Under the additional constraint that no local schedule criterion can be increased.
Model

- \( N \) organizations, where each organization \( O^{(k)} \) has \( m^{(k)} \) identical processors and \( n^{(k)} \) jobs to be executed;

- Each job \( J_i^{(k)} \) (\( 1 \leq i \leq n^{(k)} \)) requires exactly \( q_i^{(k)} \) processors for \( p_i^{(k)} \) units of time;

- Each user submits his/her own jobs locally in his/her organization.
Impact of the local constraint

- What makes this problem interesting is the additional constraint that no local schedule can be worsened if compared with the schedule that one organization can obtain by itself.

- The ratio between the optimal solution and the optimal without the local constraints is asymptotically equal to $\frac{3}{2}$. 
Previous work

- This problem was first introduced by [Pascual et al., Europar’07], that proposed an algorithm and a load-balancing heuristic called ILBA for parallel rigid jobs;

- Dutot et al. refined the algorithm and obtained a 3-approximation algorithm with tight bound for parallel rigid jobs.
Without the local constraint introduced by MOSP, this problem is equivalent to the *Multiple Strip Packing Problem*. 

- [Schwiegelshohn et al., IPDPS’08] studied this problem in the context of grid computing systems. They proposed an 3-approximation algorithm for the offline case and a 5-approximation for the online case;  

- Christina Otte and Klaus Jansen just presented their new results on this problem.
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3 Game-theoretic Model

4 Future Work
Introduction

- We are working on modeling MOSP as a non-cooperative game;

- MOSP constraint of not worsening the local objective makes the problem tricky;

- We will focus in the case where all organizations have only one machine \((m^k) = 1, 1 \leq k \leq N\).
Less jobs makes the problem easier?

- The general MOSP problem is NP-hard. Taking $N = 1$, $m^{(k)} = 2$ and $q_{i}^{(k)} = 1, (\forall i, k)$ we have the classical $P2||C_{\text{max}}$ scheduling problem;

- What if we have one machine per organization ($m^{(k)} = 1$), only 2 jobs per organization ($n^{(k)} = 2$) and sequential jobs ($q_{1}^{(k)} = q_{2}^{(k)} = 1$)?
NP-completeness

- Even with less jobs, the problem is NP-Complete in the strong sense.
- Proof: reduction from 3-PARTITION problem.

The decision problem version can be defined as follows:

**Instance:** the number $N$ of organizations, the size of all jobs $p_i^{(k)}$ and an integer $K$;

**Question:** does there exist a feasible scheduling with $C_{max} = \max_{i,k}\{p_i^{(k)}\} \leq K$?
Sketch of the proof

First, let's see how to reduce from the 2-PARTITION problem:
Sketch of the proof

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Sketch of the proof

- In the 3-PARTITION problem we want to partition a set of 3m integers (that sums up to mB) into m disjoint sets composed of exactly three elements (that sums up to B).
- To extend this proof to reduce from 3-PARTITION we must take:
  - An instance of 3-PARTITION (\{a_1, \ldots, a_{3m}\}, B), where \(\sum_{i=1}^{3m} a_i = mB\);
  - \(N = 4m\) organizations;
  - For the first 3m organizations, we set \(p_1^{(k)} = (m + 1)B + 7\) and \(p_2^{(k)} = (m + 1)a_k + 1, \forall k \in [1; 3m]\);
  - For the remaining organizations (3m + 1 to 4m), we set \(p_1^{(k)} = p_2^{(k)} = 2, \forall k \in [3m + 1; 4m]\) (the last m organizations have two jobs of size 2).
Sketch of the proof

We can build an optimal schedule for the described instance with makespan exactly equal to $(m + 1)B + 7$:

```
O(1) (m+1)B + 7 (m+1)a_1 + 1
O(2) (m+1)B + 7 (m+1)a_2 + 1
...
O(3m) (m+1)B + 7 (m+1)a_{3m} + 1
O(3m+1) 2 2
...
O(4m) 2 2
```
We can build an optimal schedule for the described instance with makespan exactly equal to \((m+1)B + 7\):
Sketch of the proof

We can build an optimal schedule for the described instance with makespan exactly equal to \((m + 1)B + 7\):

\[
\begin{align*}
O^{(1)} & \quad (m+1)B + 7 \\
O^{(3m)} & \quad (m+1)B + 7 \\
O^{(3m+1)} & \quad 2 \quad 2 \quad (m+1)a_1 + 1 \quad (m+1)a_2 + 1 \quad (m+1)a_3 + 1 \\
O^{(3m+2)} & \quad 2 \quad 2 \quad (m+1)a_4 + 1 \quad (m+1)a_5 + 1 \quad (m+1)a_6 + 1 \\
\vdots & \quad \vdots \\
O^{(4m)} & \quad 2 \quad 2 \quad (m+1)a_{3m-2} + 1 \quad (m+1)a_{3m-1} + 1 \quad (m+1)a_{3m} + 1
\end{align*}
\]
Proposed model

- We are studying a non-cooperative game defined as follows:
  - Each player is an organization responsible for an “application” (a set of $n^{(k)}$ jobs) and wants to minimize its $cost^{(k)}$ (completion time of its last job, average completion time, etc.);
  - Each organization applies some schedule algorithm locally (LPT, SPT, etc.) putting its own jobs first;
  - A strategy $S^{(k)}$ for player $k$ is a vector of $n^{(k)}$ elements such that $S^{(k)}(i)$ corresponds to the organization chosen by player $k$ for job $J_i^{(k)}$;
  - A configuration (profile) $M$ is the vector $(S^{(1)}, S^{(2)}, \ldots, S^{(N)})$ such that $S^{(k)}$ is a strategy of player $k$. 
Nash equilibrium

- A configuration \( M = (S^{(1)}, S^{(2)}, \ldots, S^{(N)}) \) is a Nash equilibrium if all players \( k \) (applications) satisfies the following property:

\[
\forall s \in S^{(k)}, \quad \text{cost}^{(k)}(M) \leq \text{cost}^{(k)}(s, M_{-k}), \text{ where } M_{-k} \text{ is a vector } (S^{(1)}, S^{(2)}, S^{(k-1)}, S^{(k+1)} \ldots, S^{(N)})
\]

- Do there always exist Nash Equilibria for MOSP\((k : C_{\max})\) or MOSP\((k : \sum C_i)\)?
Nash equilibrium and MOSP\((k : C_{\text{max}})\)

If every organization uses LPT and puts its jobs first, then there are instances of MOSP\((k : C_{\text{max}})\) where we do not have equilibrium:

\[
\begin{align*}
O^{(1)} & : 5, 4, 2 \\
O^{(2)} & : 5, 3 \\
O^{(3)} & : 1 \\
O^{(4)} & : 1
\end{align*}
\]

\[
C_{\text{max}}^{(1)} = 11 \\
C_{\text{max}}^{(2)} = 8
\]

Suppose this initial configuration.
Nash equilibrium and MOSP\((k : C_{max})\)

If every organization uses LPT and puts its jobs first, then there are instances of MOSP\((k : C_{max})\) where we do not have equilibrium:

\[
\begin{align*}
O^{(1)} & : 5 \\
O^{(2)} & : 5 \\
O^{(3)} & : 4 \\
O^{(4)} & : 3, 2
\end{align*}
\]

\[
C_{max}^{(1)} = 6 \\
C_{max}^{(2)} = 5
\]

What if \(O^{(1)}\) changes its strategy?
Nash equilibrium and MOSP\( (k : C_{\text{max}}) \)

If every organization uses LPT and puts its jobs first, then there are instances of MOSP\( (k : C_{\text{max}}) \) where we do not have equilibrium:

\[
\begin{align*}
O^{(1)} & \quad 5 \\
O^{(2)} & \quad 5 \\
O^{(3)} & \quad 1 \quad 2 \\
O^{(4)} & \quad 1 \quad 4 \quad 3 \\
\end{align*}
\]

\[ C_{\text{max}}^{(1)} = 6 \quad 5 \]
\[ C_{\text{max}}^{(2)} = 5 \quad 8 \]

What if \( O^{(2)} \) changes its strategy?
Nash equilibrium and MOSP($k : C_{max}$)

If every organization uses LPT and puts its jobs first, then there are instances of MOSP($k : C_{max}$) where we do not have equilibrium:

If $O^{(1)}$ changes its strategy?

$$C_{max}^{(1)} = 5 \ 6$$
$$C_{max}^{(2)} = 8 \ 5$$
Nash equilibrium and MOSP\( (k : C_{\text{max}}) \)

If every organization uses LPT and puts its jobs first, then there are instances of MOSP\( (k : C_{\text{max}}) \) where we do not have equilibrium:

\[
\begin{array}{c}
O^{(1)} & 5 \\
O^{(2)} & 5 \\
O^{(3)} & 4 & 3 \\
O^{(4)} & 1 & 2 \\
\end{array}
\]

\( C^{(1)}_{\text{max}} = 6 \)  \( 5 \)
\( C^{(2)}_{\text{max}} = 5 \)  \( 8 \)

What if \( O^{(2)} \) changes its strategy?
Nash equilibrium and MOSP($k : C_{max}$)

If every organization uses LPT and puts its jobs first, then there are instances of MOSP($k : C_{max}$) where we do not have equilibrium:

\[
\begin{align*}
O^{(1)} & \quad 5 \\
O^{(2)} & \quad 5 \\
O^{(3)} & \quad 1 \quad 4 \\
O^{(4)} & \quad 1 \quad 3 \quad 2 \\
\end{align*}
\]

\[
C_{max}^{(1)} = 5 \quad 6 \\
C_{max}^{(2)} = 8 \quad 5
\]

Loop!
Nash equilibrium and MOSP\((k : \sum C_i)\)

If every organization uses SPT and puts its jobs first, then there are instances of MOSP\((k : \sum C_i)\) where we do not have equilibrium:

\[
\begin{align*}
O^{(1)} & : 2 \\
O^{(2)} & : 2 \\
O^{(3)} & : 2 2 2 2 2 2 2 3 5 \\
O^{(4)} & : 2 2 2 2 2 2 2 4 6
\end{align*}
\]

\[
\sum C_i^{(3)} = 77, \quad \sum C_i^{(4)} = 98
\]

Suppose this initial configuration.
Nash equilibrium and MOSP($k : \sum C_i$)

If every organization uses SPT and puts its jobs first, then there are instances of MOSP($k : \sum C_i$) where we do not have equilibrium:

\[
\begin{align*}
O^{(1)} & : 2 & 3 & 5 \\
O^{(2)} & : 2 & 2 & 4 & 6 \\
O^{(3)} & : 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
O^{(4)} & : 2 & 2 & 2 & 2 & 2 & 2 & 2
\end{align*}
\]

\[
\sum C_i^{(3)} = 57 \\
\sum C_i^{(4)} = 68
\]

What if $O^{(4)}$ changes its strategy?
Nash equilibrium and MOSP($k : \sum C_i$)

If every organization uses SPT and puts its jobs first, then there are instances of MOSP($k : \sum C_i$) where we do not have equilibrium:

\[
\sum C_i^{(3)} = 57 \, 61 \\
\sum C_i^{(4)} = 68 \, 60
\]

What if $O^{(3)}$ changes its strategy?
Nash equilibrium and MOSP\((k : \sum C_i)\)

If every organization uses SPT and puts its jobs first, then there are instances of MOSP\((k : \sum C_i)\) where we do not have equilibrium:

\[
\begin{align*}
O^{(1)} & : \quad 2 \quad 2 \quad 5 \\
O^{(2)} & : \quad 2 \quad 3 \quad 4 \quad 6 \\
O^{(3)} & : \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \\
O^{(4)} & : \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \\
\end{align*}
\]

\[
\sum C^{(3)}_i = 61 \quad 56 \\
\sum C^{(4)}_i = 60 \quad 70
\]

What if \(O^{(4)}\) changes its strategy?
Nash equilibrium and MOSP($k : \sum C_i$)

If every organization uses SPT and puts its jobs first, then there are instances of MOSP($k : \sum C_i$) where we do not have equilibrium:

<table>
<thead>
<tr>
<th>$O^{(1)}$</th>
<th>2</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O^{(2)}$</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$O^{(3)}$</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$O^{(4)}$</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

\[
\sum C_i^{(3)} = 56 \quad 60 \\
\sum C_i^{(4)} = 70 \quad 65
\]

What if $O^{(3)}$ changes its strategy?
Nash equilibrium and MOSP\((k : \sum C_i)\)

If every organization uses SPT and puts its jobs first, then there are instances of MOSP\((k : \sum C_i)\) where we do not have equilibrium:

<table>
<thead>
<tr>
<th>Organization</th>
<th>Jobs</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>O(^{(1)})</td>
<td>2 3 4</td>
<td>6</td>
</tr>
<tr>
<td>O(^{(2)})</td>
<td>2 2 6</td>
<td>10</td>
</tr>
<tr>
<td>O(^{(3)})</td>
<td>2 2 2 2 2 2 2 2</td>
<td>16</td>
</tr>
<tr>
<td>O(^{(4)})</td>
<td>2 2 2 2 2 2 2 2</td>
<td>16</td>
</tr>
</tbody>
</table>

\[ \sum C_i^{(3)} = 60 \quad 56 \]
\[ \sum C_i^{(4)} = 65 \quad 70 \]

What if \(O^{(4)}\) changes its strategy?
Nash equilibrium and MOSP($k: \sum C_i$)

If every organization uses SPT and puts its jobs first, then there are instances of MOSP($k: \sum C_i$) where we do not have equilibrium:

![Diagram showing scheduling instances](image)

\[
\sum C_i^{(3)} = 56, 60 \\
\sum C_i^{(4)} = 70, 65
\]

Loop!
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Future work

- Study of:
  - Price of Anarchy (ratio between the worst objective function value of an equilibrium and the optimal)
  - Price of Stability (ratio between the best objective function value of one of its equilibria and the optimal outcome)

- $\epsilon$-approximate Nash Equilibrium

- Fairness